

Homework #7 (10 points) - Show all work on the following problems:

Problem 1 (2 points): Start with the following scalar and vector potentials:

$$V(\vec{r}, t) = 0, \vec{A}(\vec{r}, t) = -\frac{1}{4\pi\epsilon_0} \frac{qt}{r^2} \hat{r}$$

1a (1 point): Find the corresponding electric and magnetic fields, and the charge and current distributions that produce them.

1b (1 point): Use the gauge function $\lambda = -\frac{1}{4\pi\epsilon_0} \frac{qt}{r}$ to transform the potentials to a form which should look much more familiar.

Problem 2 (2 points): Start with the potentials $V(\vec{r}, t) = 0, \vec{A}(\vec{r}, t) = A_0 \sin(kx - \omega t) \hat{y}$, with A_0, k , and ω constants. Find the electric and magnetic fields, and check that they satisfy all four of Maxwell's equations in vacuum. What condition must be satisfied by k and ω ? Seem familiar?

Problem 3 (3 points): A point charge $q(t)$ at the origin (i.e. $\rho(\vec{r}) = q(t)\delta^3(\vec{r})$) is steadily increased by a current density $\vec{j}(\vec{r}) = -\frac{1}{4\pi r^2} \frac{dq}{dt} \hat{r}$.

3a (1 point): Use the continuity equation to check that charge is conserved.

3b (1 point): Find the scalar and vector potentials in the Coulomb gauge.

3c (1 point): Find the electric and magnetic fields, and check that they satisfy all four of Maxwell's equations in vacuum.

Problem 4 (3 points): Assume that the loop of wire at right has a current that increases linearly with time (i.e. $I(t) = kt$ for all times t). Calculate the retarded vector potential \vec{A} at the origin. Assume that the loop lies in the x-y plane with the center of curvature of the two arcs at the origin. From \vec{A} , find the electric field at the origin. Since you only know the vector potential at one point, you can't use it to find the magnetic field.

